

Non-Locality and Classical Communication of the Hidden Variable Theories

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Abstract

In all local realistic theories worked out till now, locality is considered as a basic assumption. Most people in the field consider the inconsistency between local realistic theories and quantum mechanics to be a result of non-local nature of quantum mechanics. In this paper, we derive Bell's inequality for particles with instantaneous interactions, and show that the aforementioned contradiction still exists between quantum mechanics and non-local hidden variable models. Then, we use this non-locality to obtain the GHZ theorem. In what follows, we show that Bacon and Toner's protocol, for the simulation of Bell correlation, by using local hidden variables augmented by classical communication, have some inconsistency with quantum mechanics. Our approach can answer to Brassard questions from another viewpoint, we show that if we accept that our nature obeys quantum mechanical laws, then all of quantum mechanic results cannot be simulated by realistic theories augmented by classical communication or a single instance use of non-local box.

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1 Introduction

One of the main problems in physics that has attracted physicists' attention in recent years is locality. This notion has different meanings, interpretations and applications in different fields of studies. Physicists consider locality principle as a physical constraint which should be satisfied by any new theory. Quantum mechanics (QM) has been challenging this principle for a long period. Non-locality in QM, however, enters into calculations as a consequence of the entanglement between some appropriate degrees of freedom of two separated particles, which makes them to show a correlated behavior. However, an exact quantitative relation between non-locality and entanglement has not been known. The non-local property of QM was first demonstrated by Einstein-Podolsky-Rosen (EPR) [1], who explicitly suggested that any physical theory must be both local and realistic. The manifestation of these conditions then appeared in the so-called Bell inequality [2], where locality is a crucial assumptions violated by quantum mechanical predictions. Bell's inequality has been derived in different ways [3, 4, 5]. From the so-called Bell's inequalities one can infer Bell's theorem which states that: "*In a certain experiment all realistic local theories with hidden variables are incompatible with quantum mechanics*". In Bell's theorem, the locality assumption was involved quantitatively for the first time.

Greenberger *et al.* [6], Hardy [7] and Cabello [8] have shown that it is possible to demonstrate the existence of non-locality for the case of more than two particles without using any inequality. In a recent paper, Barrett *et al.* [9] used the two-side memory loophole, in which the hidden variables of n pair can depend on the previous measurement choices and outcomes in both wings of the experiment. They have shown that the two-side memory loophole allows a systematic violation of the CHSH inequality. In another model, Scarani and Gisin [10] considered some superluminal hidden communication or influences to reproduce the non-local correlations. Various experiments have been performed to distinguish between QM and local realistic theories [11]. They all give strong indications against local realism.

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Others' works on this subject can be summarized as follows: The extension of Bell's inequality and Greenberger, Horne, Zeilinger (GHZ) theorem to continuous-variables [12]; The Bell-type inequality that involves all possible settings of the local measurement apparatus [13]; The extension of local hidden variable models (LHV) to multiparticle and multilevel systems [14]; The violation of Bell's inequality beyond Cirelson's bound [15].

Bell proved that quantum entanglement enables two space-like separated parties to exhibit classically impossible correlations. Even though these correlations are stronger than anything classically achievable, they cannot be harnessed to make instantaneous (faster than light) communication possible.

Some people extend Bell's approach, by considering realistic interpretation of QM and show that an exact simulation of the Bell correlation (singlet state) is possible by using local hidden variables augmented by just one bit of classical communication [16, 17]. Hence, C. Caves and co-works presented a model, motivated by the criterion of reality, put forward by EPR and supplemented by classical communication, which correctly reproduces the quantum mechanical predictions for measurements of all products of Pauli operators in an n -qubit GHZ state and an arbitrary graph state [18].

Hence, Popescu and Rohrlich [19] have shown that even stronger correlations can be defined, under which instantaneous communication remains impossible. It has been recently shown that all causal correlations between two parties which respectively one bit outputs a and b upon receiving one bit inputs x and y can be expressed as convex combinations of local correlations (i.e., correlations that can be simulated with local random variables) and non-local correlations of the form $a + b = x \cdot y \pmod 2$. It is also shown that a single instance of the latter elementary non-local correlation suffices to simulate exactly all possible projective measurements that can be performed on the singlet state of two qubits, with no communication needed at all [19]. Recently, G. Brassard and co-workers raised the question [16] that was repeated again by Bacon and Toner [17]: *Can we find a Bell inequality with an auxiliary communication violated by a quantum state, using a set of quantum measurements?* Hence, they consider this question from another viewpoint, and ask: Why are the correlations achievable by quantum mechanics not maximal among those that preserve causality? They give a partial answer to this question by showing that slightly stronger correlations would result in a world in which communication complexity becomes trivial [20].

In this Paper, we derive the Bell-type inequality [2] and prove a GHZ-type [6] theorem for particles with instantaneous (non-local) interactions at the hidden-variable level. Then, *we show that the previous contradiction still exists between QM and non-local hidden variable models*. In what follows, we consider model suggested by Toner and Bacon [17] and proposed again by Cerf and co-workers in the another way [19]. In the continuation, we would like to answer some questions posed by G. Brassard *et al.* [16, 20] from another viewpoint, and we show that if we accept that nature obeys quantum mechanical laws, then all of quantum mechanic all results cannot be simulated by realistic theories, augmented by classical communication [17] or a single use of nonlocal box [19].

2 Non-Locality and Hidden Variable Theory

Let us now apply this to the standard EPR-Bell setup in which a particle source with total vanishing spin ($S = 0$) emits pairs of oppositely- directed neutral spin- $\frac{1}{2}$ particles in a spin singlet state. The state vector of the system of two particles is:

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|+-\rangle_{12} - |-+\rangle_{12}), \quad (1)$$

where $|+-\rangle_{12}$ indicates that the spin of the first (second) particle in the z -basis is $+\frac{1}{2}$ ($-\frac{1}{2}$). These two particles are separated in opposite directions and when their separation becomes space-like, two experimenters (Alice and Bob), measure their spin components along \hat{a} and \hat{b} , respectively. If the result of measurement on the particle 1 (2) is called A (B), then, in a local hidden variable (LHV) model, the locality condition is defined as:

$$A(a, b, \lambda) = A(a, \lambda), \quad B(a, b, \lambda) = B(b, \lambda). \quad (2)$$

In the above expression the result of the measurement of the particle 1 (2) is independent of parameters and results of the measurement performed on the particle 2 (1). The existing contradiction between QM and LHV models, and also the violation of Bell's inequality in experiments [11] lead us to conclude that we may doubt one of the initial assumptions of Bell's inequality, i.e, locality or reality. Of course, there are

some people who believe that other assumptions and loopholes might be involved, instead of the locality and reality assumptions [9, 23].

Some generalizations of this theorem are also available, among which the GHZ theorem [6] is the most famous one. In the GHZ theorem, the assumption that the correlation functions calculated from LHV models are equal to those obtained from QM, encounters a contradiction. However, it should be noted that the GHZ-type theorems are just for n -particle systems in which $n \geq 3$. Typically in a GHZ argument, a source with zero total spin emits four spin $\frac{1}{2}$ particles such that their total state vector is:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|++--\rangle + |--++\rangle)_{1234}. \quad (3)$$

After the particles are separated from each other in a space-like manner, the spins of the particles are measured along the directions $\hat{a}, \hat{b}, \hat{c}$ and \hat{d} , respectively. If the measurement results on particles 1 through 4 are denoted by A, B, C , and D respectively, then, the locality condition in GHZ theorem will be defined as:

$$A(a, b, c, d, \lambda) = A(a, \lambda), \quad (4)$$

Similar relations hold for B, C and D . Similar to the case of Bell's theorem, the results of different measurements are independent of each other, and also independent of other parameters. Although much effort has been done to understand the role of locality and reality assumptions, due to the existence of various definitions, a conclusive source of the contradiction is hard to explore. Also, there is no clear-cut and sharp distinction between these two conditions. If we take Einstein's definition of reality (in EPR) into account, then it can be seen that it has a close relationship to locality. In other words, had we not assumed locality, speaking about realism (as Einstein considered) would have been irrelevant. In the models in which instantaneous interactions are present (such as Bohm's model), reality is essentially considered in the context of non-local influences.

In the following, we shall give up the locality assumption in LHV models and then prove the contradiction between QM and hidden variable models. To do this, let us assume that there is a deeper hidden variable level which is represented by independent variables i (for particle 1) and j (for particle 2). We denote the corresponding distribution functions of each variable by $p(i)$ and $q(j)$, respectively. We define the results $A(a, b, \lambda)$ and $B(a, b, \lambda)$ by averaging over the deeper-level variables i and j :

$$\begin{aligned} A(a, b, \lambda) &= \int p(i) di f_A(a, \lambda, i) g_A(b, \lambda, i) \\ B(a, b, \lambda) &= \int q(j) dj f_B(a, \lambda, j) g_B(b, \lambda, j) \end{aligned} \quad (5)$$

where

$$\begin{aligned} -1 &\leq f_A(a, \lambda, i), g_A(b, \lambda, i), f_B(a, \lambda, j), g_B(b, \lambda, j) \leq 1 \\ -1 &\leq A(a, b, \lambda), B(a, b, \lambda) \leq 1 \\ \int p(i) di &= \int q(j) dj = 1 \quad p(i) \geq 0 \quad q(j) \geq 0 \end{aligned} \quad (6)$$

This type of generalization is not special. Actually, for any realistic variable we can make such an assumption. Also, these variables could not be considered just a mathematical generalization but as something that could convey some physical degrees of freedom [24].

Considering the above relation, one can show that the following relation is satisfied:

$$\begin{aligned} &|f_A(a, \lambda, i) g_A(b, \lambda, i) f_B(a, \lambda, j) g_B(b, \lambda, j) - f_A(a, \lambda, i) g_A(b', \lambda, i) f_B(a, \lambda, j) g_B(b', \lambda, j)| \\ &+ |f_A(a', \lambda, i) g_A(b', \lambda, i) f_B(a', \lambda, j) g_B(b', \lambda, j) + f_A(a', \lambda, i) g_A(b, \lambda, i) f_B(a', \lambda, j) g_B(b, \lambda, j)| \leq 2. \end{aligned} \quad (7)$$

By averaging on i and j variables, we get:

$$|A(a, b, \lambda) B(a, b, \lambda) - A(a, b', \lambda) B(a, b', \lambda)| + |A(a', b, \lambda) B(a', b, \lambda) + A(a', b, \lambda) B(a', b, \lambda)| \leq 2$$

This equation tells us that every realistic model (generally as a non-local one) must satisfy the above inequality. Otherwise, we encounter an intrinsic inconsistency in the realistic model. Using this relation and averaging this time on λ , which has the distribution function $\rho(\lambda)$, we derive Bell's inequality.

Now, we can extend the above approach to derive GHZ's theorem. For the hidden variables in this theorem, the extension is as follows:

$$A(a, b, c, d, \lambda) = \int p(i) di f_A(a, \lambda, i) g_A(b, \lambda, i) h_A(c, \lambda, i) k_A(d, \lambda, i) \quad (8)$$

Similar relation can be defined for B, C and D , which are in general different from the local form of eq. (4). Thus, we have:

$$-1 \leq f_t(a, \lambda, i), g_t(b, \lambda, i), h_t(c, \lambda, i), k_t(d, \lambda, i) \leq 1 \quad t = A, B, C, D$$

After some simple algebra (similar to the case of the usual GHZ arguments) the contradiction between GHZ theorem and QM can be obtained. Likewise, This type of generalization to nonlocal cases is not restricted to eq. (5). We can generalize the above approach to arbitrary numbers of deeper hidden-variable levels to define arbitrary realistic hidden variables theories. We shall have very general cases, such as:

$$\begin{aligned} A(a, b, \lambda) &= \int p(i) di f_A(a, b, \lambda, i) g_A(a, b, \lambda, i) \\ B(a, b, \lambda) &= \int q(j) dj f_B(a, b, \lambda, j) g_B(a, b, \lambda, j) \\ f_s(a, b, \lambda, i) &= \int p'(l) dl f'_A(a, \lambda, i, l) g'_A(b, \lambda, i, l) \\ g_s(a, b, \lambda, i) &= \int q'(k) dk f'_B(a, \lambda, j, k) g'_B(b, \lambda, j, k) \quad s = A, B \end{aligned} \quad (9)$$

With arbitrary distribution functions for each variable represented by $p'(l)$ and $q'(k)$, and using the above nonlocal form, we can derive Bell's inequality again. This generalization can be extended to infinite hidden variable levels [26].

There is suggestion by [28] that if one takes Bells probability densities $\rho(\lambda)$ to be dependent on the parameters λ_a^1, λ_b^2 for Alice and Bob stations respectively, then, one cannot derive Bell's inequality. We claim that we take $\rho(\lambda)$ to be the form:

$$\rho(\lambda, \lambda_a^1, \lambda_b^2) = \int \sigma(\lambda, \lambda_a^1, \omega) \tau(\lambda, \lambda_b^2, \omega) v(\omega) d\omega \quad (10)$$

then, one can deduce Bell's inequality.

In the next section, we consider another approach, our approach is nearly similar to the GHZ theorem. We use Toner and Bacon arguments for the simulation of QM and show that although their arguments simulate quantum mechanical correlation function exactly, their approach cannot simulate all of QM results. This new approach indicates that our approach is not restricted to equations (5) or (9) and that deeper concepts are involved.

3 Bacon and Toner Simulation of Quantum Correlation function

Recently, Bacon and Toner [17] calculated classical resources required to simulate quantum correlations. They have shown that for the exact simulation of an entangled Bell pair state, we only need to use local hidden variables, augmented by just one bit of classical communication (although in this protocol, we cannot make any distinction between an instantaneous effect and classical communication). Hence, Cerf *et al.* [19], derived same results with the use of non-local machine.

In the following, we would like answer a question posed by G. Brassard *et al.* [16] and repeated again by Bacon and Toner [17]. The question is that by using a set of quantum measurements, one can find a Bell inequality with an auxiliary communication violated by a quantum state?

We consider the same protocol that was presented by Bacon and Toner. In that protocol, Alice and Bob share two random variables $\hat{\lambda}_1$ and $\hat{\lambda}_2$ which are real three dimensional unit vectors. These random variables are chosen independently and distributed uniformly over the unit sphere (infinite communication at this stage). Their protocol proceeds as follows: (1) Alice outputs $\alpha = -\text{sgn}(\hat{a} \cdot \hat{\lambda}_1)$. (2) Alice sends a single bit $c \in \{-1, +1\}$ to Bob where $c = \text{sgn}(\hat{a} \cdot \hat{\lambda}_1) \text{sgn}(\hat{a} \cdot \hat{\lambda}_2)$. (3) Bob outputs $\beta = \text{sgn}[\hat{b} \cdot (\hat{\lambda}_1 + c\hat{\lambda}_2)]$, where sgn

function is defined by $\text{sgn}(x) = +1$ if $x \geq 0$ and $\text{sgn}(x) = -1$ if $x < 0$. The joint expectation value $\langle \alpha\beta \rangle$ can be calculated using the following relation:

$$\langle \alpha\beta \rangle = E\{-\text{sgn}(\hat{a} \cdot \hat{\lambda}_1) \sum_{d=\pm 1} \frac{1+cd}{2} \times \text{sgn}[\hat{b} \cdot (\hat{\lambda}_1 + d\hat{\lambda}_2)]\}$$

where $E\{x\} = \frac{1}{(4\pi)^2} \int d\hat{\lambda}_1 \int d\hat{\lambda}_2 x$.

Here, we extend Hardy's approach [27] to non-local cases. We consider the following statements:

$$\begin{aligned} A(\hat{a}_1, \hat{\lambda}_1)B(\hat{b}_2, c(\hat{a}_1), \hat{\lambda}_1, \hat{\lambda}_2) &= -1, \\ A(\hat{a}_3, \hat{\lambda}_1)B(\hat{b}_2, c(\hat{a}_3), \hat{\lambda}_1, \hat{\lambda}_2) &= -1, \\ A(\hat{a}_3, \hat{\lambda}_1)B(\hat{b}_4, c(\hat{a}_3), \hat{\lambda}_1, \hat{\lambda}_2) &= -1, \\ A(\hat{a}_5, \hat{\lambda}_1)B(\hat{b}_4, c(\hat{a}_5), \hat{\lambda}_1, \hat{\lambda}_2) &= -1, \\ &\dots\dots\dots \\ &\dots\dots\dots \\ &\dots\dots\dots \\ A(\hat{a}_{N-1}, \hat{\lambda}_1)B(\hat{b}_N, c(\hat{a}_{N-1}), \hat{\lambda}_1, \hat{\lambda}_2) &= -1, \\ A(\hat{a}_1, \hat{\lambda}_1)B(\hat{b}_N, c(\hat{a}_1), \hat{\lambda}_1, \hat{\lambda}_2) &= +1. \end{aligned} \tag{11}$$

where N is even. Similar to Hardy's arguments, we define the probability $p^\pm(\hat{a}, \hat{b})$ for getting the result of $A(\hat{a}, \hat{\lambda}_1)B(\hat{a}, \hat{b}, \hat{\lambda}_1, \hat{\lambda}_2) = \pm 1$. Then, the probabilities for each of statements (11) being true are, respectively:

$$\begin{aligned} p_1^- &= p^-(\hat{a}_1, \hat{b}_2), \\ p_2^- &= p^-(\hat{a}_3, \hat{b}_2), \\ p_3^- &= p^-(\hat{a}_3, \hat{b}_4), \\ p_4^- &= p^-(\hat{a}_5, \hat{b}_4), \\ &\dots\dots\dots \\ &\dots\dots\dots \\ &\dots\dots\dots \\ p_{N-1}^- &= p^-(\hat{a}_{N-1}, \hat{b}_N), \\ p_N^+ &= p^+(\hat{a}_1, \hat{b}_N). \end{aligned} \tag{12}$$

Hardy [27] defined the probability P to be true for all the statements (12). The probability that one or more of the statements to be false must be less than or equal to the sum of the probabilities for each individual statement to be false, *i.e.*:

$$1 - P \leq \sum_{n=1}^{N-1} (1 - p_n^-) + (1 - p_N^+)$$

After some simple algebra, he derived the following Bell's inequality:

$$\sum_{n=1}^{N-1} p_n^- + p_N^+ > N - 1 \tag{13}$$

In the above equation p_n^- and p_N^+ represent the probability that each individual statement (11) is true. If we consider the singlet state (1) and calculate the prediction of quantum mechanics, we get:

$$p^\pm(a, b) = \frac{1}{2}[1 \mp \cos(a - b)]$$

Hardy has chosen the angles $a_1, b_2, a_3, \dots, b_N$ to be evenly spread, so that:

$$b_N - a_1 = \phi, \quad b_n - a_{n\pm 1} = \mp \frac{\phi}{N-1} \tag{14}$$

The maximum violation of the inequality (13) is obtained for:

$$\phi = \frac{(N-1)\pi}{N} \quad (15)$$

He has also shown that for $N \rightarrow \infty$, the probability that all the statements of (11) are true tends to 1 ($P \rightarrow 1$). In other words, for any measurement of two-particle systems all the statements (11) are correct.

If we insert Alice and Bob outputs in the equation (11), all the statements change to:

$$\begin{aligned} -\text{sgn}(\hat{a}_{2i+1} \cdot \hat{\lambda}_1) \left[\frac{1-c(\hat{a}_{2i+1})}{2} \text{sgn}(b_{2i} \cdot (\hat{\lambda}_1 - \hat{\lambda}_2)) + \frac{1+c(\hat{a}_{2i+1})}{2} \text{sgn}(b_{2i} \cdot (\hat{\lambda}_1 + \hat{\lambda}_2)) \right] &= -1, \\ & i = 1, \dots, \frac{N}{2} - 1 \\ -\text{sgn}(\hat{a}_{N-1} \cdot \hat{\lambda}_1) \left[\frac{1-c(\hat{a}_{N-1})}{2} \text{sgn}(b_N \cdot (\hat{\lambda}_1 - \hat{\lambda}_2)) + \frac{1+c(\hat{a}_{N-1})}{2} \text{sgn}(b_N \cdot (\hat{\lambda}_1 + \hat{\lambda}_2)) \right] &= -1, \\ -\text{sgn}(\hat{a}_1 \cdot \hat{\lambda}_1) \left[\frac{1-c(\hat{a}_1)}{2} \text{sgn}(b_N \cdot (\hat{\lambda}_1 - \hat{\lambda}_2)) + \frac{1+c(\hat{a}_1)}{2} \text{sgn}(b_N \cdot (\hat{\lambda}_1 + \hat{\lambda}_2)) \right] &= +1. \end{aligned}$$

If we multiply l.h.s. and r.h.s., we obtain:

$$\begin{aligned} \left\{ \prod_{i=1}^{\frac{N}{2}-1} \prod_{k=\pm 1} \left[\frac{1-c(\hat{a}_{2i+k})}{2} \text{sgn}(b_{2i} \cdot (\hat{\lambda}_1 - \hat{\lambda}_2)) + \frac{1+c(\hat{a}_{2i+k})}{2} \text{sgn}(b_{2i} \cdot (\hat{\lambda}_1 + \hat{\lambda}_2)) \right] \right\} \quad (16) \\ \times \left[\frac{1-c(\hat{a}_{N-1})}{2} \text{sgn}(b_N \cdot (\hat{\lambda}_1 - \hat{\lambda}_2)) + \frac{1+c(\hat{a}_{N-1})}{2} \text{sgn}(b_N \cdot (\hat{\lambda}_1 + \hat{\lambda}_2)) \right] \\ \times \left[\frac{1-c(\hat{a}_1)}{2} \text{sgn}(b_N \cdot (\hat{\lambda}_1 - \hat{\lambda}_2)) + \frac{1+c(\hat{a}_1)}{2} \text{sgn}(b_N \cdot (\hat{\lambda}_1 + \hat{\lambda}_2)) \right] = -1 \end{aligned}$$

If we choose $\hat{\lambda}_1 = -\hat{\lambda}_2$ (noting the Toner and Bacon protocol, Alice and Bob have unlimited access to $\hat{\lambda}_1$ and $\hat{\lambda}_2$), we get $c(\hat{a}) = -1$ (for all statements). Thus, the product of all these equations must be equal to +1 on the l.h.s., and equal to -1 on the r.h.s.. As we said, for $N \rightarrow \infty$, all statements of equation (11) must be true for all $\hat{\lambda}_1, \hat{\lambda}_2$. Thus, one gets an inconsistency between Bacon and Toner's protocol and quantum mechanics.

Before finishing this section, we would like to explain some points. Our extension can be considered not only for classical communication but also for instantaneous effects. Only in the explanation of protocol, we have a difference between an instantaneous interaction and a classical communication, but at the mathematical level, we have no difference. In other words, we can consider an instantaneous effect that changes Bob's results as mentioned before. Recently, this non-local approaches was suggested by Cerf and co-workers, who derived the Bacon and Toner results with the replacement of classical communications with a single instance of the non-local machine [19].

4 Discussion

Bell's theorem [2] states that any local realistic view of the world is incompatible with QM, while, this is often interpreted as demonstrating the existence of non-locality in QM [25]. There exist some types of models that simulate quantum correlation function. In these models, quantum correlation function simulate exactly all possible projective measurements that can be performed on the singlet state of two qubits, by using local hidden variables augmented by just one bit of classical communication or by a single instance of the non-local machine (without any communication needed at all). The amounts of classical communication (one instance of nonlocal Popescu-Rohrlich machine) has been considered as the amount of the non-locality inherent in quantum mechanics at entanglement in the singlet state. Although, these theories explain some part of QM, but they could not proposes a complete description of QM. For example, in the non-local machine, we have not all of QM properties. It has been shown that entanglement swapping are not simulated by non-local machine [21] and quantum multiparties correlation arising from measurements on a cluster state cannot be simulated with n non-local boxes, for any n [22].

By following these works, G. Brassard and co-workers have propounded an important questions [16]: have a Bell inequality been fund with auxiliary communication that is violated by a quantum state and a set of quantum measurements? Hence, they given responses to two other important questions [20]: (1)

Considering that perfect nonlocal box's would not violate causality, why do the laws of quantum mechanics only allow us to implement nonlocal box's better than anything classically possible, yet not perfectly? (2) Why do they provide us with an approximation of nonlocal box's that succeed with probability $\wp = 85.4$ rather than something better? They give a partial answer to this questions by showing that in any world in which communication complexity is nontrivial, there is a bound on how much nature can be nonlocal. This bound, which is an improvement over the previous knowledge that nonlocal boxes could not be implemented exactly, approaches the actual bound $\wp = 85.4$, imposed by quantum mechanics. The obvious open question is to close the gap between these probabilities. A proof that nontrivial communication complexity forbids nonlocal boxes to be approximated with probability greater than \wp , would be very interesting, as it would render Tsirelson's bound [29] inevitable, making it a candidate for a new information-theoretic axiom for quantum mechanics.

We have responded to G. Brassard and co-workers important questions [20] in another way. We have considered hidden variable theories augmented by classical communication [17] or a single use of nonlocal box [19] and have shown that these theories cannot simulate QM (not Bell's correlations) by local hidden variables augmented by classical communication or by nonlocal effects. We don't claim that Toner and Bacon protocol is wrong - that protocol simulates Bell's correlation functions *exactly* - but it isn't enough. In other words, *it cannot simulate all of QM results without internal inconsistency*.

In this Paper, we have considered hidden variable theories with instantaneous (non-local) interactions and have shown that similar to local hidden variable theories, there exists an incompatibility between QM and non-local hidden variables theories. Hence, we have answered Brassard questions from another viewpoint, we have shown that if we accept that nature obeys quantum mechanical laws, then all of quantum mechanical results cannot be simulated by realistic theories augmented by classical communication or a single use of nonlocal box.

Therefore, it can be concluded that some other alternative view points might be involved. Some people conclude that the assumption of the existence of a reality independent of observation may be irrelevant to physics [30]. On the other hand, some people still believe that QM is a local theory [31], and some others consider information the root of the interpretation of QM [32]. Another non-local hidden variable model is Bohmian quantum mechanics (BQM) [33] (although, this theory does not provide a complete theory for the spin variable). Could one apply our extension to BQM? Anyway, the above argument indicates that we must have a deeper understanding of the notions of locality, reality and entanglement.

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